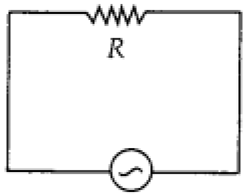
7. Alternating Current

Question 1.
A 100 Q resistor is connected to a 220 V, 50 Hz a.c. supply.
(a) What is the rms value of current in the circuit?
(b) What is the net power consumed over a full cycle?
Solution:
(a) Here virtual a.c. voltage is 220 V at a frequency of 50 Hz. So, rms value of current

$$I_v = \frac{E_v}{R} = \frac{220}{100} = 2.2 \text{ A}$$



(b) Power in complete cycle

$$P = E_v I_v \cos\phi = E_v I_v \cos 0°$$

$$P = 2.2 \times 220 = 484 \text{ W}$$

Question 2.
(a) The peak voltage of an a.c. supply is 300 V. What is the rms voltage?
(b) The rms value of current in an a.c. circuit is 10 A. What is the peak current?
Solution:
(a) The peak value of a.c. supply is given 300 V.

$E_0 = 300$ V

So, rms value of voltage

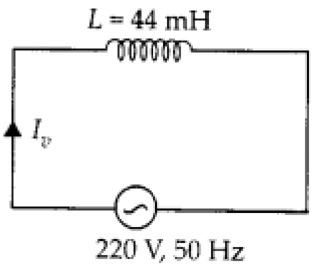
$$E_v = \frac{E_0}{\sqrt{2}} = \frac{300}{\sqrt{2}} = 150\sqrt{2} \text{ V} = 212.1 \text{ V}$$

(b) Here $I_v = 10$ A

Thus, peak current $I_0 = I_v\sqrt{2} = 10\sqrt{2}$ A $= 14.1$ A

Question 3.
A 44 mH inductor is connected to 220 V, 50 Hz ac supply. Determine the rms value of current in the circuit.

Solution:



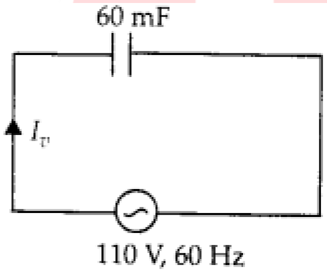
The rms current is

$$I_v = \frac{E_v}{X_L} = \frac{220}{2\pi \times 50 \times 44 \times 10^{-3}} = 15.9 \text{ A}$$

Question 4.

A 60 μF capacitor is connected to a 110 V, 60 Hz a.c. supply. Determine the rms value of current in the circuit.

Solution:



Capacitive reactance $X_C = \frac{1}{2\pi f C}$

$$X_C = \frac{1}{2 \times \pi \times 60 \times 60 \times 10^{-6}} = 44.2 \ \Omega$$

The rms current is

$$I_v = \frac{E_v}{X_C} = \frac{110}{44.2} = 2.5 \text{ A}$$

Question 5.

In previous questions 3 and 4, what is the net power absorbed by each circuit over a complete cycle. Explain your answer.

Solution:

For question 3, Power in the circuit with pure inductor P = Eυlυ cos π/2 = 0. For question 4, Power in complete cycle P = Eυlυ cos (-π/2) = 0.

Question 6.
Obtain the resonant frequency a), of a series LCR circuit with L = 2.0 H, C = 32 μF and R = 10 Ω2. What is the Q-value of this circuit?
Solution:
Resonant angular frequency in series LCR circuit

$$\omega_r = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{2 \times 32 \times 10^{-6}}} = 125 \text{ rad/sec}$$

$$\text{Quality factor } Q = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{10}\sqrt{\frac{2}{32 \times 10^{-6}}} = 25.$$

Question 7.
A charged 30 pF capacitor is connected to a 27 mH inductor. What is the angular frequency of free oscillations of the circuit?
Solution:
Angular frequency of LC oscillations

$$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{27 \times 10^{-3} \times 30 \times 10^{-6}}}$$

$$\omega = \frac{1}{9 \times 10^{-4}} = \frac{1}{9} \times 10^4 \text{ rad/sec} = 1.1 \times 10^3 \text{ rad/sec}$$

Question 8.
Suppose the initial charge on the capacitor given in question 7 is 6 mC. What is the total energy stored in the circuit initially? What is the total energy at later time?
Solution:
Initial energy on capacitor

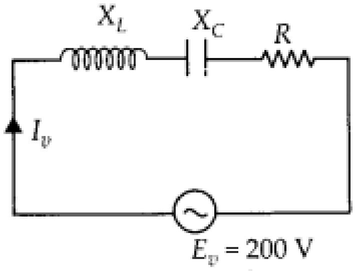
$$U_E = \frac{q_0^2}{2C} = \frac{\left(6 \times 10^{-3}\right)^2}{2 \times 30 \times 10^{-6}} = 0.6 \text{ J}$$

Any time total energy in the circuit is constant, hence energy later is 0.6 J.

Question 9.
A series LCR circuit with R = 20 Ω 2, L = 1.5 H and C = 35 μF is connected to a

variable-frequency 200 V ac supply. When the frequency of the supply equals the natural frequency of the circuit, what is the average power transferred to the circuit in one complete cycle?

Solution:



Average power transferred to the circuit in one complete cycle at resonance

$$P = E_v I_v \cos\phi$$

$$P = E_v \frac{E_v}{Z} \cos\phi$$

At resonance $Z = R$, $\cos\phi = \cos 0° = 1$

$$P = 200 \times \frac{200}{20} = 2000 \text{ W}$$

Question 10.

A radio can tune over the frequency range of a portion of MW broadcast band: (800 kHz to 1200 kHz), if its LC circuit has an effective inductance of 200 μH, what must be the range of its variable capacitor?

Solution:

For tuning, the natural frequency i.e., the frequency of L-C oscillations should be equal to frequency of radio waves received by the antenna in the form of same frequency current in the L-C circuit. For tuning at 800 kHz, required capacitance

$$f_1 = \frac{1}{2\pi\sqrt{LC_1}}$$

$$C_1 = \frac{1}{4\pi^2 L f_1^2} = \frac{1}{4\pi^2(200\times10^{-6})(800\times10^3)^2}$$

$$= 197.8 \text{ pF}$$

For tuning of 1200 kHz, required capacitance

$$f_2 = \frac{1}{2\pi\sqrt{LC_2}}$$

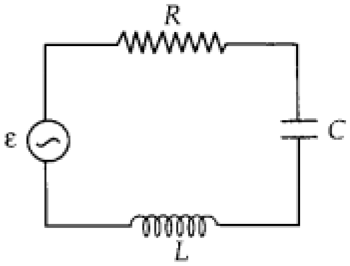
$$C_2 = \frac{1}{4\pi L f_2^2} = \frac{1}{4\pi^2(200\times10^{-6})(1200\times10^3)^2}$$

$$= 87.9 \text{ pF}$$

So, the variable capacitor should have a frequency range between 87.9 pF to 197.8 pF.

Question 11.

Figure shows a series LCR circuit connected to a variable frequency 230 V source. L = 5.0 H. C = 80 μF, ft = 40 Q



(a) Determine the source frequency which drives the circuit in resonance.
(b) Obtain the impedance of the circuit and the amplitude of current at the resonating frequency.
(c) Determine the rms potential drops across -the three elements of the circuit. Show that the potential drop across the LC combination is zero at the resonating

Solution:

(a) Condition for resonance is when applied frequency matches with natural frequency.

Resonant frequency $\omega_r = \frac{1}{\sqrt{LC}}$

$$= \frac{1}{\sqrt{5(80 \times 10^{-6})}} = 50 \text{ rad s}^{-1}$$

(b) At resonance, impedance Z = R

as $X_L = X_C$

So, $Z = 40\ \Omega$

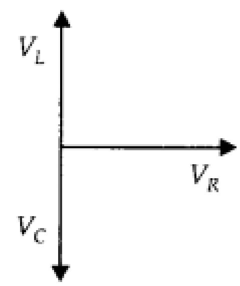
rms current, $I_v = \frac{E_v}{R} = \frac{230}{40} = 5.75$ A

Amplitude of current, $I_0 = I_v\sqrt{2} = 8.13$ A

(c) Potential drop across 'L'

$V_L = I_v X_L = 5.75 \times (\omega L)$

$V_L = 5.75 \times 50 \times 5 = 1437.5$ V



Potential drop across 'C'

$V_C = I_v \times X_C = 5.75 \times \frac{1}{\omega C}$

$= 5.75 \times \frac{1}{50 \times 80 \times 10^{-6}}$

$= 1437.5$ V

Potential drop across R

$V_R = I_v R = 5.75 \times 40 = 230$ V

As $V_L - V_C = 0$, So $E_v = V_R = 230$ V.

Question 12.

An LC circuit contains a 20 mH inductor and a 50 μF capacitor with initial charge of 10 mC. The resistance of the circuit in negligible. Let the instant the circuit is closed be t = 0.

(a) What is the total energy stored initially? Is it conserved during LC oscillations?

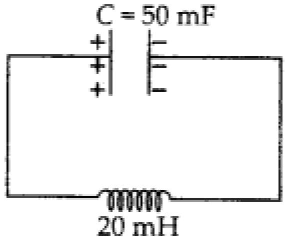
(b) What is the natural frequency of the circuit?

(c) At what time is the energy stored

- completely electrical (i.e., stored in the capacitor)?
- completely magnetic (i.e., stored in the inductor)?

(d) At what times is the total energy shared equally between the inductor and capacitor?
(e) If a resistor is inserted in the circuit, how much energy is eventually dissipated as heat?

Solution:



(a) Total energy is initially in the form of electric field within the plates of charged capacitor.

$$U_E = \frac{Q^2}{2C} = \frac{(10\times10^{-3})^2}{2\times50\times10^{-6}} = 1\text{ J}$$

If we neglect the losses due to resistance of connecting wires, the total energy remain consumed during LC oscillations.

(b) Natural frequency of the circuit

$$f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\times\sqrt{20\times10^{-3}\times50\times10^{-6}}}$$

$$= \frac{500}{\pi}\text{ Hz} = 159\text{ Hz}$$

(c) Instantaneous electrical energy

$$U_E = \frac{q_0^2 \cos^2 \omega t}{2C}$$

At $\omega t = 0, \pi, 2\pi, 3\pi\ldots$ the energy is completely electrical.

$$t = \frac{n\pi}{2\pi f} = \frac{n}{2f} = \frac{n\pi}{1000} \text{ sec; } n = 0, 1, 2, 3, 4$$

or $t = 0, T/2, T, 3T/2 \ldots$

Instantaneous magnetic energy

$$U_B = \frac{1}{2} L q_0^2 \omega^2 \sin^2 \omega t \quad \text{or} \quad U_B = \frac{q_0^2}{2C} \sin^2 \omega t$$

so at $\omega t = \pi/2, 3\pi/2, 5\pi/2\ldots$

The energy is completely magnetic

$$t = \frac{(2n+1)\pi}{2(2\pi f)} = \frac{(2n+1)}{4f} = \frac{(2n+1)\pi}{2000} \text{ sec}$$

where $n = 0, 1, 2, 3, 4 \ldots$

or $t = T/4, 3T/4, 5T/4\ldots$

(d) timings for energy shared equally between inductor and capacitor.

$U_B = U_E$

$$\frac{q_0^2}{2C} \sin^2 \omega t = \frac{q_0^2}{2C} \cos^2 \omega t$$

$\tan^2 \omega t = 1$ or $\tan \omega t = \tan \pi/4$

$$t = \frac{\pi}{4\omega}, \frac{3\pi}{4\omega}, \frac{5\pi}{4\omega} \ldots \quad \text{or} \quad t = \frac{T}{8}, \frac{3T}{8}, \frac{5T}{8} \ldots$$

(e) When a resistor is inserted in the circuit, eventually all the energy will be lost as heat across resistance. The oscillation will be damped.

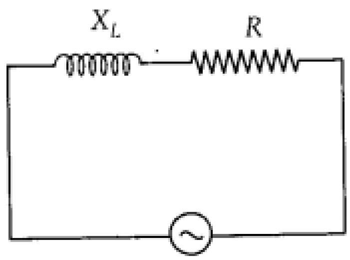
Question 13.

A coil of inductance 0.50 H and resistance 100 Ω2 is connected to a 240 V. 50 Hz ac supply.

(a) What is the maximum current in the coil?

(b) What is the time lag between the voltage maximum and the current maximum?

Solution:



Inductive reactance

$X_L = 2\pi f L = 2\pi \times 50 \times 0.5 = 157\ \Omega$

Impedence $Z = \sqrt{R^2 + X_L^2} = \sqrt{(100)^2 + (157)^2}$

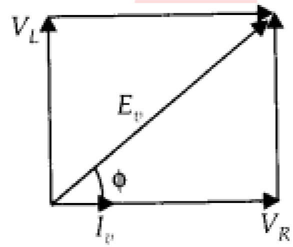
$= 186.14\ \Omega$

(a) Virtual current in the coil

$$I_v = \frac{E_v}{Z} = \frac{240}{186.14} = 1.29\ \text{A}$$

Maximum current, $I_0 = I_v\sqrt{2} = 1.82$ A

(b)



Phase lag, $\tan\phi = \frac{X_L}{R} = 1.57$

$\phi = \tan^{-1}(1.57) = 57.5°$ or $\phi = 0.32\,\pi$ radian

Time lag, $t = \phi/\omega = 3.2$ ms

Question 14.

Obtain the answers (a) and (b) in Q. 13, if the circuit is connected to a high frequency supply (240 V, 10 kHz). Hence, explain the statement that at very high frequency, an inductor in a circuit nearly amounts to an open circuit. How does an inductor behave in a dc circuit after the steady state?

Solution:

At very high frequency, X_L increases to infinitely large, hence circuit behaves as open circuit.

$X_L = 2\pi f L = 2\pi(10 \times 10^3) \times 0.5 = 31400\ \Omega$

(a) Current in the coil, $I_{rms} = \varepsilon_{ms} Z$

Maximum current in the coil,

$$I_0 = \sqrt{2}\, I_{rms} = \sqrt{2} \times \frac{\varepsilon_{rms}}{Z}$$

$$= 1.414 \times \frac{240\ \text{V}}{3.14 \times 10^4\ \Omega}\ \text{A} = 1.10 \times 10^{-2}\ \text{A}$$

This current is extremely small. Thus, at high frequencies, the inductive reactance of an inductor is so large that it behaves as an open circuit.

(b)

$$\text{As } \tan\phi = \frac{X_L}{R} = \frac{3.14 \times 10^4}{100} = 314,\ \phi \approx 90^\circ$$

$$\text{Clearly, time lag} = \frac{90^\circ}{360^\circ} \times \frac{1}{10^4}\ \text{s} = 25 \times 10^{-6}\ \text{s}$$

In dc circuit (after steady state), v = 0 and as such $X_L = 0$. In this case, the inductor behaves like a pure resistor as it has no inductive reactance.

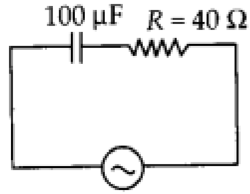
Question 15.

A 100 μF capacitor in series with a 40 Q resistance is connected to a 110 V, 60 Hz supply.

(a) What is the maximum current in the circuit?

(b) What is the time lag between the current maximum and the voltage maximum?

Solution:



$$\text{Capacitive reactance, } X_C = \frac{1}{2\pi f C}$$

$$X_C = \frac{1}{2 \times \pi \times 60 \times 100 \times 10^{-6}} = 26.54\ \Omega$$

$$\text{Impedence, } Z = \sqrt{R^2 + X_C^2}$$

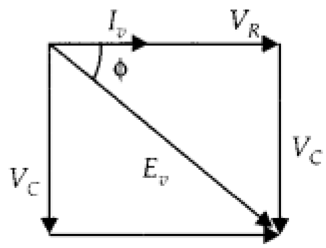
$$= \sqrt{(40)^2 + (26.54)^2} = 48\ \Omega$$

(a) Virtual current in the coil

$$I_v = \frac{E_v}{Z} = \frac{110}{48} = 2.29 \text{ A}$$

Maximum current $I_0 = I_v\sqrt{2} = 3.24$ A

(b)



$$\tan\phi = \frac{V_C}{V_R} = \frac{1}{\omega CR}$$

Phase lag $\phi = \tan^{-1}\left(\frac{1}{\omega CR}\right) = \tan^{-1}\left(\frac{26.54}{40}\right)$

$\phi = 33.56° = 0.186\pi$ radian

Time lag $t = \phi/\omega = \frac{0.18\pi}{2\pi(60)} = 1.5$ ms

Question 16.

Obtain the answer to (a) and (b) in Q.15 if the circuit is connected to a 110 V, 12 kHz supply? Hence, explain the statement that a capacitor is a conductor at very high frequencies. Compare this behaviour with that of a capacitor in a dc circuit after the steady state.

Solution:

Given,

$E_{rms} = 110$ V, $\upsilon = 12$ kHz $= 12 \times 10^3$ Hz

(a)

$$X_c = \frac{1}{2 \times 3.14 \times (12 \times 10^3) \times 10^{-4}} = 0.1326\ \Omega$$

As, $R = 40\ \Omega$, $X_c <<< R$

$Z \approx R = 40\ \Omega$

$$I_{rms} = \frac{\varepsilon_{rms}}{Z} = \frac{110}{40} = 2.75 \text{ A}$$

$$I_0 = \sqrt{2}\, I_{rms} = 1.414(2.75) = 3.9 \text{ A}$$

This value of current is same as that without capacitor in the circuit. So, at high frequency, a capacitor offer negligible resistance (0.1326 Ω in this case), it behave like a conductor.

(b)

$$\text{As } \tan\phi = \frac{X_C}{R} = \frac{0.1326\ \Omega}{40\ \Omega} = 0.0033,$$

$$\phi = 0.189° \approx 0°$$

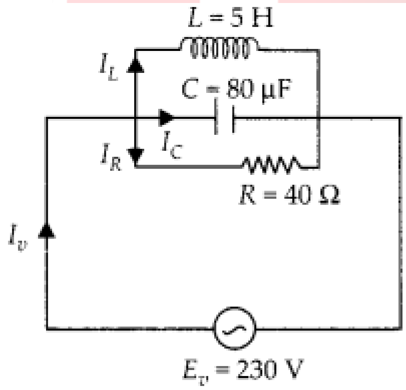
$$\text{Time lag} = \frac{0.189°}{360°} \times \frac{1}{12 \times 10^3} = 43.8 \times 10^{-9}\ \text{s}$$

In dc circuit, after steady state, v = 0 and accordingly, $X_C = \infty$, i.e., a capacitor amounts to an open circuit, i.e., it is a perfect insulator of current.

Question 17.

Keeping the source frequency equal to the resonating frequency of the series LCR circuit, if the three elements, L, C and R are arranged in parallel, show that the total current in the parallel LCR circuit is minimum at this frequency. Obtain the current rms value in each branch of the circuit for the elements of frequency. Source has emf 230 V and L = 5.0 H, C = 80 μF, ff = 40 Ω2.

Solution:



Resonating angular frequency

$$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{5 \times 80 \times 10^{-6}}} = 50\ \text{rad s}^{-1}$$

∴ Resonance of L and C in parallel can be calculated

$$\frac{1}{X} = \frac{1}{X_L} - \frac{1}{X_C} = \frac{1}{\omega L} - \omega C$$

impedance of R and X in parallel is given by

$$\frac{1}{Z} = \sqrt{\frac{1}{R^2} + \frac{1}{X^2}}$$

At resonating frequency of series *LCR*, $X_L = X_C$

So, $\frac{1}{X} = \frac{1}{X_L} - \frac{1}{X_C} = 0$

Thus, impedance Z = R and will be maximum. Hence, in parallel resonant circuit, current is minimum at resonant frequency. Current through circuit elements

$$I_R = \frac{E_v}{R} = \frac{230}{40} = 5.75 \text{ A}$$

$$I_C = \frac{E_v}{X_L} = \frac{230}{\omega L} = \frac{230}{50 \times 5} = 0.92 \text{ A}$$

$$I_C = \frac{E_v}{X_L} = \frac{230}{(1/\omega C)}$$

$$= 230 \times 50 \times 80 \times 10^{-6} = 0.92 \text{ A}$$

Since, I_L and I_C are opposite in phase, so net current,

$$I_v = I_R + I_L + I_C$$

$$I_v = 5.75 + 0.92\sqrt{2}\sin(\omega t - \pi/2) + 0.92\sqrt{2}\sin(\omega t + \pi/2)$$

$$I_v = 5.75 - 0.92\sqrt{2}\cos\omega t + 0.92\sqrt{2}\cos\omega t$$

$$I_v = 5.75 \text{ A}$$

Question 18.

A circuit containing a 80 mH inductor and a 60 μF capacitor in series is connected to a 230 V, 50 Hz supply. The resistance of the circuit is negligible.

(a) Obtain the current amplitude and rms values.

(b) Obtain the rms values of potential drops across each element.

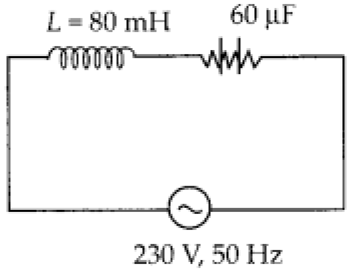
(c) What is the average power transferred to the inductor?

(d) What is the average power transferred to the capacitor.

(e) What is the total average power absorbed by the circuit?

['Average'implies'averaged over one cycle'.

Solution:



(a) Inductive reactance , $X_L = 2\pi f L$

$X_L = 2\pi(50)\, 80 \times 10^{-3} = 25.12\ \Omega$

Capacitive reactance, $X_C = \dfrac{1}{2\pi f C}$

$$X_C = \frac{1}{2\times 3.14\times 50\times 60\times 10^{-6}} = 53.05\ \Omega$$

Impedance $= X_C - X_L = 53.05 - 25.12 = 27.93\ \Omega$

rms value of current, $I_v = \dfrac{E_v}{Z} = \dfrac{230}{27.93} = 8.235$ A

Peak value $I_0 = I_v\sqrt{2} = 11.644$ A

(b) Potential drop across L, $V_L = I_v X_L$ X

$= 206.68$ V

Potential drop across C, $V_C = I_v X_C = 436.87$ V

(c) Average power transferred to inductor is zero, because of phase difference π/2

$P = E_v I_v \cos\phi$

$\phi = \pi/2, \quad \therefore \quad P = 0$

(d) Average power transferred to capacitor is also zero, because of phase difference π/2

$P = E_v I_v \cos\phi$

$\phi = \pi/2, \quad \therefore \quad P = 0$

(e) total power absorbed by the circuit

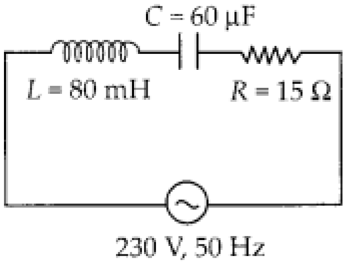
$P_{Total} = P_L + P_C = 0$

Question 19.

Suppose the circuit in Q. 18 has a resistance of 15 Ω2. Obtain the average power transferred to each element of the circuit and the total power absorbed.

Solution:

If the circuit has a resistance of 15 Ω2, now it is LCR series resonant circuit.



Now the impedance,

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$Z = \sqrt{15^2 + (27.93)^2} = 31.7\ \Omega$$

Virtual current, $I_v = \frac{E_v}{Z} = \frac{230}{31.7} = 7.26$ A

Average power transferred to 'L',

$P_L = I_v E_v \cos \pi/2 = 0$

Average power transferred to 'C',

$P_C = E_v I_v \cos \pi/2 = 0$

Average power transferred to 'R',

$P_R = V_R I_v \cos 0°$

$P_R = (I_v R)\, I_v = I_v^2\, R = (7.26)^2 \times 15 = 791$ W

Question 20.

A series LCR circuit with L = 0.12 H, C = 480 μF, R = 23 Ω2 is connected to a 230 V variable frequency supply.

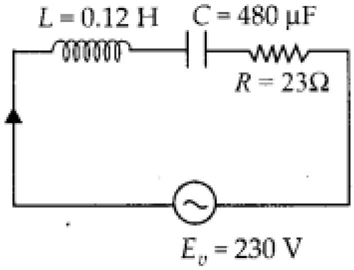
(a) What is the source frequency for which current amplitude is maximum. Obtain this maximum value.

(b) What is the source frequency for which average power absorbed by the circuit is maximum. Obtain the value of this maximum power.

(c) For which frequencies of the source is the power transferred to the circuit half the power at resonant frequency? What is the current amplitude at these frequencies?

(d) What is the Q-factor of the given circuit?

Solution:



(a) At resonant frequency, the current amplitude is maximum.

$$f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.12\times480\times10^{-9}}} = 663\text{ Hz}$$

$$I_v = \frac{E_v}{R},\ I_0 = I_v\sqrt{2} = \frac{E_v\sqrt{2}}{R} = \frac{230\sqrt{2}}{23} = 14.14\text{ A}$$

(b) Maximum power loss at resonant frequency, P = Eυlυ cos θ

frequency, $P = E_v I_v \cos\phi$

$$P = E_v\frac{E_v}{R}\cos 0° = \frac{E_v^2}{R} = \frac{(230)^2}{23} = 2300\text{ W}$$

(c) Let at an angular frequency, the source power is half the power at resonant frequency.

$P = E_v I_v \cos\phi$

$$\frac{1}{2}\left[\frac{E_v^2}{R}\right] = \frac{E_v E_v}{Z} \cdot \frac{R}{Z}$$

$Z^2 = 2R^2$

$R^2 + (X_L - X_C)^2 = 2R^2$

$X_L - X_C = R$

$$\omega_L - \frac{1}{\omega C} = R \quad \text{or} \quad \omega^2 - \frac{1}{LC} = \frac{R}{L}\omega$$

where resonant angular frequency

$$\omega_r = \frac{1}{LC} = \frac{1}{0.12 \times 480 \times 10^{-9}}$$

so, $\omega^2 - \omega_r^2 = \pm \frac{R}{L}\omega$

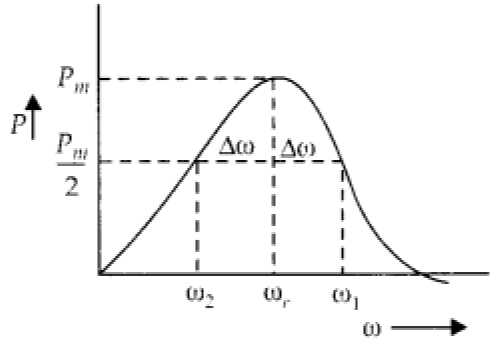
two quadratic equations can be formed

$$\omega^2 - \frac{R}{L}\omega - \omega_r^2 = 0 \quad \text{and} \quad \omega^2 + \frac{R}{L}\omega - \omega_r^2 = 0$$

On solving, we get

$$\omega_1 = \frac{R}{2L} + \left[\omega_r^2 + R^2/4L^2\right]^{1/2} = \omega_r + \Delta\omega \text{ and}$$

$$\omega_2 = -\frac{R}{2L} + \left[\omega^2 + \frac{R^2}{4L^2}\right]^{1/2} = \omega_r - \Delta\omega$$



Now, $\omega_1 - \omega_2 = R/L$

$[\omega_r + \Delta\omega] - [\omega_r - \Delta\omega] = R/L$ or $\Delta\omega = R/L$

$\Delta\omega = \frac{R}{2L}$ bandwidth of angular frequency

So, band width of frequency

$$\Delta f = \frac{\Delta\omega}{2\pi} = \frac{R}{4\pi L} = \frac{23}{4 \times 3.14 \times 0.12}$$

$\Delta f = 15.26$ Hz

Hence the two frequencies for half power

$f_Z = f_2 - \Delta f$ and $f_1 = f_r + \Delta f$

$f_Z = 663 - 15.26 = 647.74$ Hz

and $f_1 = 663 + 15.26 = 678.26$ Hz

At these frequencies the current amplitude is

$$I = \frac{I_0}{\sqrt{2}} = 10 \text{ A}$$

(d) Q factor, $Q = \frac{1}{R}\sqrt{\frac{L}{C}}$

$$Q = \frac{1}{23}\sqrt{\frac{0.12}{480 \times 10^{-9}}} = 21.7$$

Question 21.

Obtain the resonant frequency and Q-factor of a series LCR circuit with L = 3.0 H, C= 27 μF, and R = 7.4 fl. It is desired to improve the sharpness of the resonance of the circuit by reducing its 'full width at half maximum' by a factor of 2. Suggest a suitable way.

Solution:

$$\upsilon_r = \frac{1}{2\pi\sqrt{LC}}$$

$$= \frac{1}{2 \times 3.14 \times \sqrt{3 \times 27 \times 10^{-6}}} = \frac{1000}{6.28 \times 9} = 17.7$$

resonant frequency, $\omega_r = 2\pi\upsilon_r$

$= 2 \times 3.14 \times 17.7$

$= 111$ rad sec^{-1}

Quality factor in the given resonant circuit

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{7.4}\sqrt{\frac{3}{27 \times 10^{-6}}} = 45$$

we want to improve the quality factor to twice, without changing resonant frequency (without changing L and C).

$$Q' = 2Q = 90 = \frac{1}{R'}\sqrt{\frac{L}{C}}$$

$$\text{or} \quad R' = \frac{1}{90}\sqrt{\frac{3}{27\times10^{-6}}} = 3.7\ \Omega$$

Question 22.

Answer the following questions.

(a) In any ac circuit, is the applied instantaneous voltage equal to the algebraic sum of the instantaneous voltages across the series elements of the circuit? Is the same true for rms voltage?

(b) A capacitor is used in the primary circuit of an induction coil.

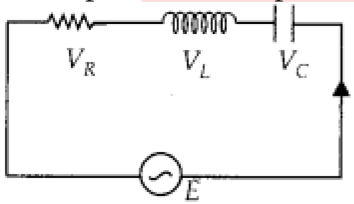
(c) An applied voltage signal consists of a superposition of a dc voltage and an ac voltage of high frequency. The circuit consists of an inductor and a capacitor in series. Show that the dc signal will appear across Cand the ac signal across L.

(d) A choke coil in series with a lamp is connected to a dc line. The lamp is seen to shine brightly. Insertion of an iron core in the choke causes no change in the lamp's brightness. Predict the corresponding observations if the connection is to an ac line.

(e) Why is choke coil needed in the use of fluorescent tubes with ac mains? Why can we not use an ordinary resistor instead of the choke coil?

Solution:

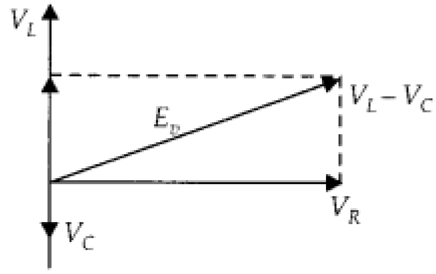
(a) It is true that applied instantaneous voltage is equal to algebraic sum of instantaneous potential drop across each circuit element is series.



$$E = V_R + V_L + V_C$$

$$E_0\sin\omega t = \frac{E_0}{R}\sin\omega t + \frac{E_0}{X_L}\sin(\omega t - \pi/2) + \frac{E_0}{X_C}\sin(\omega t + \pi/2)$$

But the rms voltage applied is equal to vector sum of potential drop across each element, as voltage drops are in different phases.



$$E_v = \sqrt{(V_L - V_C)^2 + V_R^2}$$

(b) At the time of broken circuit of the induction coil, the induced high voltage charges the capacitor. This avoid sparking in the circuit.

(c) Inductive reactance, $X_L = 2\pi f L$ For a.c., $X_c \propto f$

For d.c., $f = 0$, $X_L = 0$

Capacitive reactance, $X_C = \dfrac{1}{2\pi f C}$

For a.c., $X \propto \dfrac{1}{f}$

For d.c., $f = 0$, $X_C = \infty$

So, superimpose applied voltage will have all d.c. potential drop across Xc and will have most of a.c potential drop across XL.

(d) Inductor offer no hinderance to d.c. $X_L = 0$, so insertion of iron core does not effect the d.c. current or brightness of lamp connected. But it definitely effect a.c. current as insertion of iron core increases L = μm nl thus increases X_L ($2\pi f L$). A.c. current in the E circuit reduces $I_P = E_v X_L$ and brightness of the bulb also reduces.

(e) A fluorescent tube is connected directly across a 220 V source, it would draw large current which may damage the filaments of the tube. So a choke coil which behaves as L-R circuit reduces the current to appropriate value, and that also with a lesser power loss.

$$I_v = \frac{E_v}{\sqrt{R^2 + X_L^2}}, \quad P = E_v I_v \cos\phi$$

An ordinary esistor used to control the current would have maximum power wastage as heat.

$$I_v = \frac{E_v}{R}, \quad P_{max} = E_v I_v$$

Question 23.

A power transmission line feeds input power at 2300 V to a step down transformer with its primary windings having 4000 turns. What should be the number of turns in the secondary in order to get output power at 230 V?

Solution:

Here $E_p = 2300$ V, $N_p = 4000$ turns,
$E_s = 230$ V, $N_s = ?$

We know in a transformer, $\frac{E_s}{E_p} = \frac{N_s}{N_p}$

$$N_s = \frac{E_s N_p}{E_p} = \frac{230 \times 4000}{2300} = 400 \text{ turns}$$

Question 24.

At a hydroelectric power plant, the water pressure head is at a height of 300 m and the water flow available is 100 $m^3 s^{-1}$. If the turbine generator efficiency is 60%, estimate the electric power available from the plant ($g = 9.8\ ms^{-2}$).

Solution:

Work done by liquid pressure = pressure x volume shifted power of flowing water

$$\text{Hydro-power} = \frac{\text{Work}}{\text{time}} = \text{pressure} \times \frac{\text{volume}}{\text{time}}$$

Hydro-power = $h\rho g \times (V/t)$

$= 300 \times 10^3 \times 9.8 \times 100 = 29.4 \times 10^7$ Watt

$$\text{Efficiency of turbine } \eta = \frac{\text{Eletric power}}{\text{Hydro-power}}$$

$$0.6 = \frac{\text{Eletric power}}{29.4 \times 10^7}$$

Electric power $= 0.6 \times 29.4 \times 10^7 = 176.4 \times 10^6$ W

$= 176.4$ MW

Question 25.

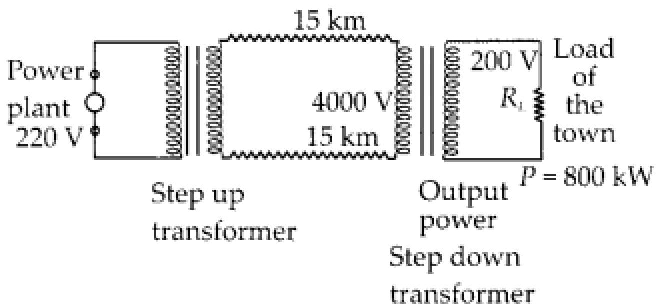
A small town with a demand of 800 kW of 1 electric power at 220 V is situated 15 km away from an electric plant generating power at 440V. The resistance of the two wire line carrying power is 0.5 Q per km. The town gets 1 power from the line through a 4000-220 V step- down transformer at a sub station in the town.

(a) Estimate the line power loss in the form of heat.

(b) How much power must the plant supply, assuming there is negligible power loss due to leakage?

(c) Characterize the step up transformer at the plant.

Solution:



Line resistance = length of two wire line × resistance per unit length

Line resistance $(R) = 2 \times 15\text{ km} \times 0.5\ \frac{\Omega}{\text{km}} = 15\ \Omega$

Virtual a.c. in the line, $P = E_v I_v$

$800 \times 10^3 = 4000\ I_V$ or $I_v = 200$ A

(a) Line power loss,

$P_{loss} = I_v^2 R = (200)^2 \times 15$
$= 600$ kW

(b) Assuming no power loss due to leakage, total power need to be supply by the power plant

$P_{total} = P_{loss} + P_{output} = 600\text{ kW} + 800\text{ kW} = 1400\text{ kW}$

(c) Potential drop in the line,

$V = I_v R = 200 \times 15 = 3000$ V

So, the voltage output of step-up transformer at the plant should be

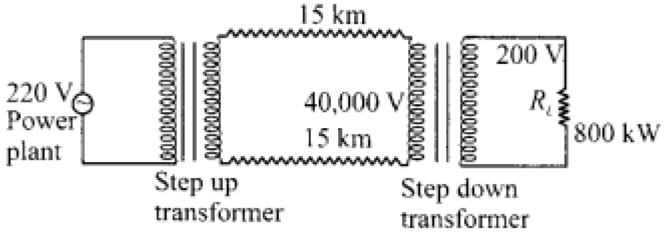
$4000 + 3000 = 7000$ V.

hence at the plant the step-up transformer should be 440 – 7000 V.

Question 26.

Repeat the same exercise as in the previous question with the replacement of the earlier transformer by a 40,000-220 V step down transformer. (Neglect, as before, leakage losses through this may not be a good assumption any longer because of the very high voltage transmission involved). Hence, explain why high voltage transmission is preferred?

Solution:



Virtual a.c in the time

$$I_v = \frac{P_{output}}{E_v} = \frac{800 \times 10^3}{40,000} = 20 \text{ A}$$

(a) Line power loss,

$P_{loss} = I_v^2 R = (20)^2 \times 15$

$= 6$ kW

(b) Power supplied by the plant

$P_{Total} = P_{Loss} + P_{output} = 6 \text{ kW} + 800 \text{ kW}$

$= 806$ kW

(c) Voltage drop in the line,

$V = I_v R = 20 \times 15 = 300$ V.

Voltage output of step-up transformer at power station = 40,000 + 300 = 40,300 V

So, the step up transformer at the power plant is 220 V-40,300 V.

Power loss in earlier arrangement,

$$P_1 = \frac{600 \times 10^3}{1400 \times 10^3} \times 100 = 43\%$$

Power loss in this arrangement,

$$P_2 = \frac{6 \times 10^3}{806 \times 10^3} \times 100 = 0.74\%$$

So, by supply of electricity at higher voltage, 40,000 V instead by 4000 V the power loss is reduced greatly that is why the electric power is always transmitted at very high voltage.